Title: Real World Systems of Linear Equations

Brief Overview:

In this unit, students will be learning, practicing, and discussing ways of representing real world situations by using systems of linear equations. The student will be able to interpret and discuss the differences between Slope-Intercept Form and Standard Form of Linear Equations, in an attempt to better understand when to use which form to represent different situations.

NCTM Content Standard/National Science Education Standard:

- write equivalent forms of equations and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases.
- use symbolic algebra to represent and explain mathematical relationships.
- be able to represent and analyze mathematical situations and structures using algebraic symbols. They will be able to use mathematical models to represent given real world situations.
- be able to build new mathematical knowledge through problems solving. The will be able to solve problems that arise in real world situations.
- be able to communicate their thinking coherently and clearly to peers, teachers, and others. The will be able to use the language of mathematics to express mathematical ideas precisely.

These lessons are also written in accordance with the Maryland State Department of Education standard Core Learning Goal 1.2.3.

- Core Learning Goal 1.2.3: The student will solve and describe using numbers, symbols, and/or graphs if and where two straight lines intersect.

Grade/Level:

Grades 9 – 12: Pre-Algebra, Algebra 1, Consumer Math

Duration/Length:

Three 50–minute sessions

Student Outcomes:

Students will be able to:

- Interpret and solve real world problems using a narrative and two linear equations in slope intercept form.
- Interpret and solve real world problems using a narrative and two linear equations in standard form
- Discern between real world standard form and real world slope intercept problems using a narratives, system characteristics and written systems.

Materials and Resources:

- Concentric Circles with Systems of Linear Equations Activity (Directions and required resources)
- Systems of Linear Equations Index Cards
- "Slope-Intercept Linear System Real World Problems" Worksheet and Answer Key
- "Where's the Money?" Worksheet and Answer Key
- "Candy is Coming Activity!!" (This has directions, student worksheet, and an answer key.)
- 6 M&M Bags and 6 Hershey Bars for each student or group
- "Standard Form Real World Model Problems" Worksheet and Answer Key
- "Warm Up Activity" Worksheet and Answer Key
- "Mix-and-Match Real World Systems of Equations" Activity (This has directions, resources, and an answer key.)
- Packet of Index cards

Development/Procedures:

Lesson 1

Preassessment – Students will complete the Concentric Circles with Systems of Linear Equations Activity. Directions and resources are located with this activity. This activity will determine whether the students know the different solutions that can be present for Systems of Linear Equations and activates their prior knowledge of solving the systems.

Launch – Teacher and students will briefly review how a single slope – intercept equation could be used to model a real world problem. Particular emphasis will be given to the real world meanings of slope as a rate of change and y – intercept as the initial condition or starting value of y. Questions are asked on the "Slope-Intercept Linear System Real World Problems" worksheet. Students will be asked to consider if a real world situation could be modeled using two slope intercept form equations.

Teacher Facilitation – Teacher will review basic verbal problem solving skills. Students will be asked to read aloud the problems on the "Slope-Intercept Linear System Real World Problems" worksheet. The teacher will elicit oral responses from the students on variable definitions, equation set ups, what the problem asks for, and techniques for solving systems of equations.

Student Application – Following completion of the first problem, students will complete a second similar problem on their own using the "Slope-Intercept Linear System Real World Problems" worksheet. The teacher will circulate through the class looking for students who need assistance and take questions.

Embedded Assessment – Teachers assess student learning through responses to verbal questions and visual observations of student work. Assessment emphasis will be placed on each student's second problem performance.

Reteaching/Extension –

- Students demonstrating sufficient skill command will be tasked to create a similar real world problem of their own. Adequate command should include the ability to set up a system and fundamental system solving skills. Small errors

in equation solving should not be considered an issue for mastery.

- Students who do not demonstrate sufficient skill command will complete a third written exercise with teacher guidance and assistance as needed. Difficulty in setting up a system or fundamental problems with solving a system would indicate insufficient skill command.
- Instruction is differentiated for the two groups.

Lesson 2

Preassessment – Students will complete the "Where's the Money" worksheet to refresh the previous lesson's skills.

Launch – Student's will complete the "Candy is Coming Activity!!!" In this activity, the students will complete a chart to determine combinations of two different items that can be purchased given a specific number of M&M Bags; they are to determine the number of Hershey Bars that fits the scenario. The activity seeks to guide students into considering real world linear equations in standard form.

Teacher Facilitation - Teacher will review basic verbal problem solving skills. Students will be asked to read aloud the problems on the "Standard Form Real World Model Problems" worksheet. The teacher will elicit oral responses from the students on variable definitions, equation set ups, what the problem asks for, and techniques for system and equation solving. All exercises come from the worksheet.

Student Application - Following completion of the first problem, students will complete a second similar problem on their own. The teacher will circulate through the class looking for students who need assistance and take questions.

Embedded Assessment - Teachers assess student learning through responses to verbal questions and visual observations of student work. Assessment emphasis will be placed on each student's second problem performance.

Reteaching/Extension

- Students demonstrating sufficient skill command will be tasked to create a similar real world problem on their own. Adequate command should include the ability to set up a system and fundamental system solving skills. Small errors in equation solving should not be considered an issue for mastery.
- Students who do not demonstrate sufficient skill command will complete a third written exercise with teacher guidance and assistance as needed. Difficulty in setting up a system or fundamental problems with solving a system would indicate insufficient skill command.
- Instruction is differentiated for the two groups.

Lesson 3

Preassessment – Students will complete the "Warm Up Activity" worksheet about the characteristics of real world slope intercept and standard form linear systems. Students will seek to develop a way to distinguish between the two types systems in a verbal problem.

Launch – Students will complete the "Mix-and-Match Real World Systems of Equations Activity." Directions and resources are located with this activity. This activity will determine whether the students know how to match real world scenarios with an appropriate System of Linear Equations.

Teacher Facilitation - Teacher will check to see that students matched the systems and real world scenario correctly and then direct students to solve the problem with their partner.

Application – Each pair of students will solve their systems problem and prepare a presentation to be delivered to the rest of the class. As the presentations take place, the other students should be taking notes, asking questions, and offering suggestion for improvement. If you have a larger class, select certain groups to give their presentations. Or you may want to do a Group Merry-Go-Round Activity in which each group prepares a poster, places the poster on the wall, and the students walk around the room placing their ideas and/or concerns on the poster.

Estimated time: 10 minutes for finding the solution and preparing presentation. Additional time is needed for presentations, based on the number of groups.

Embedded Assessment - Teacher assesses student learning through visual observation of student performance in matching the systems and real world scenario correctly and their ability to present their problem to the class.

- Students write their solutions to the system on the blackboard and present them to the class. Presentation should include definition of variables, steps to solutions and explanation of how the systems were identified.
- Students will observe and critique each other's work for clarity and correctness.

Summative Assessment:

The summative assessment will have the students complete the two types of real world problems that we concentrated on in this unit. The hope is that the students will be able to prove that they understand how to translate Real World situations into Systems of Linear Equations, and then know how to solve those Systems to determine the answer to the situation.

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Concentric Circles with Systems of Linear Equations Activity
Teacher Notes

Materials:

- Copy of the "Systems of Equations Index Cards"
- Prepared Set of Index Cards, one card for each student in the class
- Full Copy of this Activity, one for just the teacher.

Preparation:

- Prepare the index cards. For each index card, place the question on the front and the correct answer on the back. You can find questions attached to this activity.
- Determine the directions that you will want to deliver to your students. The Student directions listed below will not work for all students, because lower level students may need more direction than upper level students.

Student Directions:

- Students will get in two concentric circles. The inside circle will be facing out and the outside circle will be facing in. The hope is that the students will be facing each other in pairs.

- With their partner, one student will raise the index card with the answer facing themselves. The other student is to answer the question "How many Solutions does this system have?" (To make the activity harder you can have the students determine the solutions to each of these systems, either in the circle or when they sit down at their desks.)

- Once the first question is answered, the roles reverse.

- Once both students have answered the question, you will direct one of the circles to move, so that each student is paired up with a different student.

- Repeat the above process until you decide to stop or you run out of pairing options.

This Activity was modified from "*Cooperative Learning & Mathematics High School Activities,*" Kagan Publishing, 2001. The original Activity is called "Inside-Outside Circle."

Systems of Linear Equations Index Cards

Question Side	Answer Side
$y = 2x$ $y = -\frac{3}{5}x$	One solution $(0,0)$
$y = x + 2$ $y = 3x$	One solution $(1,3)$
$y = -2x + 1$ $y = -x - 3$	One solution $(4,-7)$
$y = -5x + 1$ $y = 3x + 1$	One solution $(0,1)$

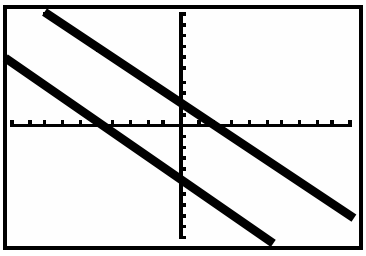
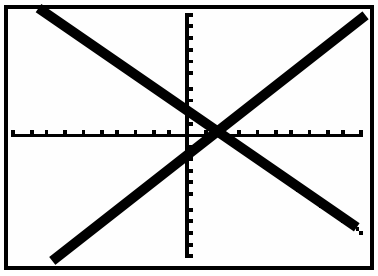
Systems of Linear Equations Index Cards

$y = \frac{x}{2} + 3$ $y = 2x - 3$	One solution $(4, 5)$
2 lines with different slopes	One Solution
$y = 3x - 2$ $y = 3x - 2$	Infinite solution points
Two lines that have the same equation	Infinite solution points

Systems of Linear Equations Index Cards

$y = \frac{2}{5}x + 6$ $y = 0.4x + 6$	Infinite solution points
Two lines that have the same slope and the same y-intercept	Infinite solution points
$y = -2x + 7$ $3y = -6x + 21$	Infinite solution points
$y = 4x + 3$ $y = 4x + 8$	No Solution

Systems of Linear Equations Index Cards

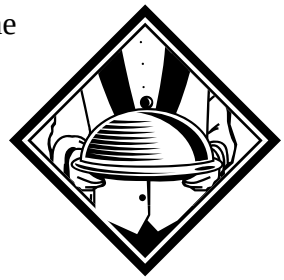
$y = \frac{1}{2}x - 3$ $y = 0.5x + 2$	No Solution
$3y = 9x + 6$ $y = 3x + 4$	No Solution
	No Solution
	One Solution

Slope-Intercept Linear System Name: _____
Real World Problems Date: _____

Complete the following sentences:

- The slope intercept form is _____
- In the real world slope, *m*, means _____ and the *y* – intercept, *b*, means _____.

1. Wendy is starting a catering business and is attempting to figure out who she should be using to transport the food to different locations. She has found two trucking companies that are willing to make sure her food arrives intact. Peter's Pick Up charges $0.40 per mile and charges a flat fee of $68. Helen's Haulers charges $0.65 per mile and charges a flat fee of $23.



   - Define your variables.
   - Write a system of equations to model the above situation.
   - For what distance would the cost of transporting to the produce be the same for both companies? What is that equal cost? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.
   - Which company charges a lower fee for a 160 mile trip? Use mathematics to justify your answer.
   - Which company will move a greater distance for $200? Use mathematics to justify your answer.

2. Jonas needs a cell phone. He has a choice between two companies with the following monthly billing policies. Each company's monthly billing policy has an initial operating fee and charge per minute.

	Operating Fee	Charge per Minute
Terri's Telephone	29.95	0.14
Carrie's Connection	4.95	0.39

- Define your variables.

- Write a system of equations to model the above situation.

- At how many minutes is the monthly cost the same? What is the equal monthly cost of the two plans? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

- Which plan costs more 150 minutes of calls each month? Use mathematics to justify your answer.

- Which plan provides more minutes for $ 60.00? Use mathematics to justify your answer.

If you felt as though you got #1 and 2 correct, go to Problem #4.
If you feel as though you need extra help go to Question 3 and do not complete Question #4.

3. Movies Are Us has two video rental plans. The Regular video rental plan charges $ 3.25 for each video rental. The Preferred video rental plan has an $ 8.75 membership fee and charges $ 2 for each video rental.

    - Define your variables.

    - Write a system of equations to model the above situation.

    - How many video rentals give the two plans the same cost? What is the equal cost? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

    - Which video plan costs more for 18 video rentals? Use mathematics to justify your answer.

    - Which plan provides more videos for $ 30.00? Use mathematics to justify your answer

4. Instead of completing another problem, be creative and write your own scenario. Be sure to give your solution as well. (Hint: The easiest way to come up with this is to determine your answer first.)

Slope-Intercept Linear System
Real World Problems

Name: *Answer Key*
Date: _____

Complete the following sentences:

- The slope intercept form is $y = mx + b$.

- In the real world slope, *m*, means the rate of change and the *y* – intercept, *b*, means the initial starting value.

1. Wendy is starting a catering business and is attempting to figure out who she should be using to transport the food to different locations. She has found two trucking companies that are willing to make sure her food arrives intact. Peter's Pick Up charges $ 0.40 per mile and charges a flat fee of $68. Helen's Haulers charges $ 0.65 per mile and charges a flat fee of $23.

- Define your variables.

  $x =$ number of miles the truck drove
  $y =$ total cost of transporting the food

- Write a system of equations to model the above situation.

  Peter's Pick Up: $y = 0.40x + 68$
  Helen's Haulers: $y = 0.65x + 23$

- For what distance would the cost of transporting to the produce be the same for both companies? What is that equal cost? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

$$0.65x + 23 = 0.40x + 68$$
$$-0.40x \qquad -0.40x$$
$$0.25x + 23 = 68$$
$$-23 \quad -23$$
$$\frac{0.25x}{0.25} = \frac{45}{0.25}$$
$$x = 180$$

$y = 0.40x + 68$
$y = 0.40(180) + 68$
$y = 72 + 68$
$y = 140$

or

$y = 0.65x + 23$
$y = 0.65(180) + 23$
$y = 117 + 23$
$y = 140$

At 180 miles, the cost for the two companies will both be $140.

- Which company charges a lower fee for a 160 mile trip? Use mathematics to justify your answer.

160 miles $\Rightarrow x = 160$

Peter's Pick Up	Helen's Haulers
$y = 0.40x + 68$	$y = 0.65x + 23$
$y = 0.40(160) + 68$	$y = 0.65(160) + 23$
$y = 64 + 68$	$y = 104 + 23$
$y = 132$	$y = 127$
\$132 for 160 miles	\$127 for 160 miles

Helen's Haulers is \$5 cheaper for 160 miles.

- Which company will move a greater distance for \$200? Use mathematics to justify your answer.

\$200 $\Rightarrow y = 200$

Peter's Pick Up	Helen's Haulers
$200 = 0.40x + 68$	$200 = 0.65x + 23$
$-68 \quad -68$	$-23 \quad -23$
$\frac{132}{0.40} = \frac{0.40x}{0.40}$	$\frac{177}{0.65} = \frac{0.65x}{0.65}$
$x = 330$	$x = 272.3$
330 miles for \$200.	272.3 miles for \$200.

Peter's Pick up will give you 57.7 more miles for the \$200.

2. Jonas needs a cell phone. He has a choice between two companies with the following monthly billing policies. Each company's monthly billing policy has an initial operating fee and charge per minute.

	Operating Fee	**Charge per Minute**
Terri's Telephone	29.95	**0.14**
Carrie's Connection	4.95	0.39

- Define your variables.

$x =$ number of minutes used for one month

$y =$ total cost for one month of service

- Write a system of equations to model the above situation.

  Terri's Telephone: $y = 0.14x + 29.95$

  Carrie's Connection: $y = 0.39x + 4.95$

- At how many minutes is the monthly cost the same? What is the equal monthly cost of the two plans? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

$$0.14x + 29.95 = 0.39x + 4.95$$
$$-0.14x \qquad -0.14x$$
$$29.95 = 0.25x + 4.95$$
$$-4.95 \qquad -4.95$$
$$\frac{25}{0.25} = \frac{0.25x}{0.25}$$
$$x = 100$$

$y = 0.14x + 29.95$
$y = 0.14(100) + 29.95$
$y = 14 + 29.95$
$y = 43.95$

or

$y = 0.39x + 4.95$
$y = 0.39(100) + 4.95$
$y = 39 + 4.95$
$y = 43.95$

At 100 minutes, both companies will cost $43.95.

- Which plan costs more 150 minutes of calls each month? Use mathematics to justify your answer.

150 minutes $\Rightarrow x = 150$

Terri's Telephone	Carrie's Connection
$y = 0.14x + 29.95$	$y = 0.39x + 4.95$
$y = 0.14(150) + 29.95$	$y = 0.39(150) + 4.95$
$y = 21 + 29.95$	$y = 58.5 + 4.95$
$y = 50.95$	$y = 63.45$
$50.95 for 150 minutes.	$63.45 for 150 minutes.

Terri's Telephone is $12.50 cheaper for 150 minutes.

- Which plan provides more minutes for $ 60.00? Use mathematics to justify your answer.

$\$60.00 \Rightarrow y = 60.00$ or $y = 60$

Terri's Telephone

$60 = 0.14x + 29.95$

$-29.95 \qquad -29.95$

$\frac{30.05}{0.14} = \frac{0.14x}{0.14}$

$x = 214.64$

214 minutes for $60.

Carrie's Connection

$60 = 0.39x + 4.95$

$-4.95 \qquad -4.95$

$\frac{55.05}{0.39} = \frac{0.39x}{0.39}$

$x = 141.15$

141 minutes for $60.

Terri's Telephone provides 73 more minutes for $60.

If you felt as though you got #1 and 2 correct, go to Problem #4.
If you feel as though you need extra help go to Question 3 and do not complete Question #4.

3. Movies Are Us has two video rental plans. The Regular video rental plan charges $ 3.25 for each video rental. The Preferred video rental plan has an $ 8.75 membership fee and charges $ 2 for each video rental.

- Define your variables.

$x =$ number of videos rented

$y =$ total cost

- Write a system of equations to model the above situation.

Regular Video Rental Plan: $y = 3.25x$

Preferred Video Rental Plan: $y = 2x + 8.75$

- How many video rentals give the two plans the same cost? What is the equal cost? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

$$3.25x = 2x + 8.75$$
$$-2x \quad -2x$$
$$\frac{1.25x}{1.25} = \frac{8.75}{1.25}$$
$$x = 7$$

$$y = 3.25x$$
$$y = 3.25(7)$$
$$y = 22.75$$

or

$$y = 2x + 8.75$$
$$y = 2(7) + 8.75$$
$$y = 14 + 8.75$$
$$y = 22.75$$

At 6 video rentals, both Rental Plans will cost $22.75.

- Which video plan costs more for 18 video rentals? Use mathematics to justify your answer.

$18 \text{ videos} \Rightarrow x = 18$

Regular Plan

$y = 3.25x$

$y = 3.25(18)$

$y = 26$

18 Rentals will cost $26.

Prefered Plan

$y = 2x + 8.75$

$y = 2(18) + 8.75$

$y = 36 + 8.75$

$y = 44.75$

18 rentals will cost $44.75.

For 18 Rentals, the Regular Video Rental Plan is $18.75 cheaper.

- Which plan provides more videos for $100.75? Use mathematics to justify your answer.

$\$100.75 \Rightarrow y = 100.75$

Regular Plan

$$\frac{100.75}{3.25} = \frac{3.25x}{3.25}$$

$x = 31$

31 video rental for $100.75.

Preferred Plan

$$100.75 = 2x + 8.75$$
$$-8.75 \quad -8.75$$
$$\frac{92}{2} = \frac{2x}{2}$$

$x = 46$

46 video rentals for $100.75.

The Preferred Video Rental Plan will get 15 more rentals for $100.75.

4. Instead of completing another problem, be creative and write your own scenario. Be sure to give your solution as well. (Hint: The easiest way to come up with this is to determine your answer first.)

Answers will vary based on what the students choose to do.

Name: _____
Date: _____

Where's the Money?

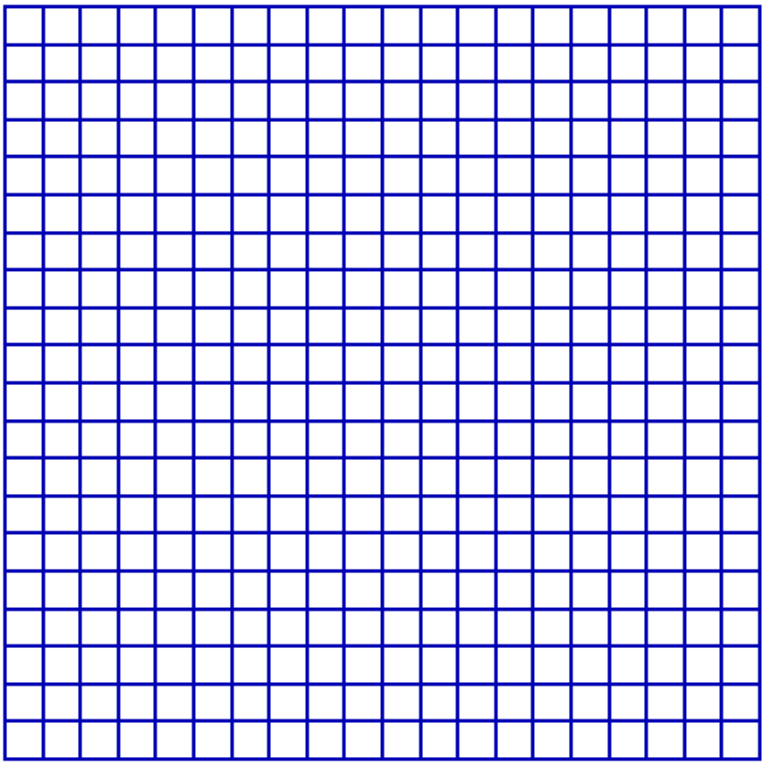
Sergeant Major Carter decided to open a JROTC uniform cleaning service at West Point High School. When he started the business he had to purchase an Ironing Board for $15 and an Iron for $35. Also, he figured it would cost $1.25 in cleaning products for each uniform, so he decided he was going to charge $3.25 for cleaning and pressing one entire uniform.

1. Write a system of equations that would represent the above scenario.

2. How many uniforms does the Sergeant Major have to clean in order to break even? Show your Work. If you choose to use the graphing method, please use the grid provided.

3. If the Sergeant Major cleans fewer uniforms than what you determined in #3, what does it mean?

4. If the Sergeant Major cleans more uniforms than what you determined in #3, what does it mean?



Name: *Answer Key*
Date: _____

Where's the Money?

Sergeant Major Carter decided to open a JROTC uniform cleaning service at West Point High School. When he started the business he had to purchase an Ironing Board for $15 and an Iron for $35. Also, he figured it would cost $1.25 in cleaning products for each uniform, so he decided he was going to charge $3.25 for cleaning and pressing one entire uniform.

1. Write a system of equations that would represent the above scenario.

   Let x = the number of uniforms cleaned
   Let y = the total cost

   y = 50 + 1.25x or *y = 1.25x + 50*
   y = 3.25x *y = 3.25x*

2. How many uniforms does the Sergeant Major have to clean in order to break even? Show your Work. If you choose to use the graphing method, please use the grid provided.

   Method 1:
   $$1.25x + 50 = 3.25x$$
   $$-1.25x \quad -1.25x$$
   $$\frac{50}{2} = \frac{2.00x}{2}$$
   $$25 = x$$

   Sergeant Major Carter must clean 25 uniforms to break even.
   Justification: $50 + 1.25x = 50 + 1.25*25 = \$81.25$
   $3.25x = 3.25*25 = \$81.25$

   Method 2:
   Student puss the two equations in on the grid provided and determine eht point of intersection to be (25, 81.25).

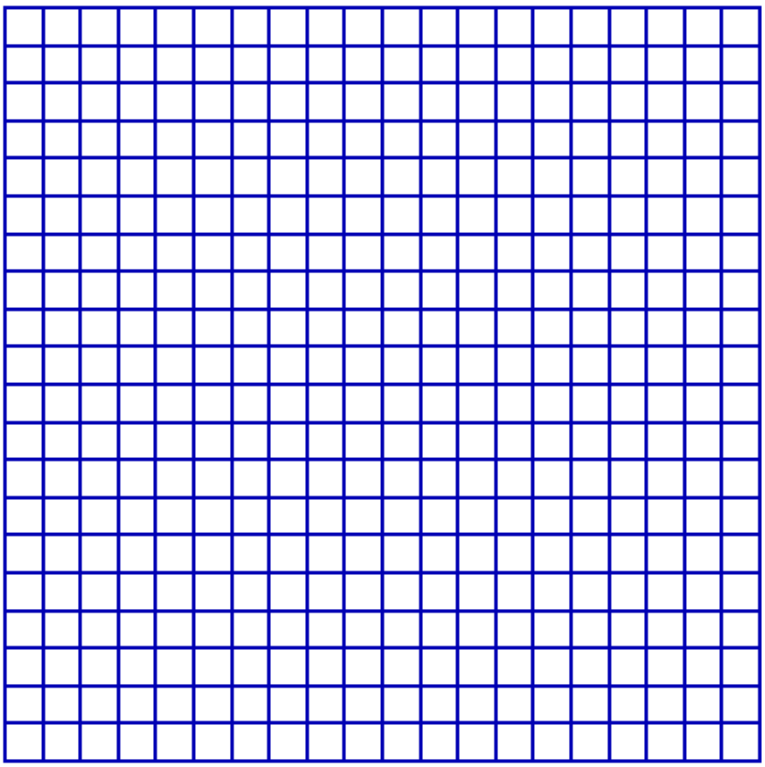
   Method 3:
   Student puts the two equations in y_1 and y_2 of the TI-83/84, then goes to the Table and scrolls until the values in y_1 and y_2 are equal. This occurs when x = 25. When x = 25, both y values are $81.25.

3. If the Sergeant Major cleans fewer uniforms than what you determined in #3, what does it mean?

If the Sergeant Major cleans fewer than 25 uniforms, than that means he has not met the Break Even point and is still losing money on his endeavor.

4. If the Sergeant Major cleans more uniforms than what you determined in #3, what does it mean?

If the Sergeant Major cleans more than 25 uniforms, than that means he has met the Break Even point and is now making money on his endeavor.



Candy is Coming Activity!!!

Teacher Notes

Materials:

- "Candy is Coming!!!" Worksheet, one for each student
- Six M&M Bags and six Hershey Bars, one for each student or group
- Full Copy of this Activity, one for just the teacher.

Teacher Preparation:

- Decide whether you want the students to complete this activity individually or in groups.
- Prepare enough Bags of M&Ms and Hershey Bars. Each item must have the correct purchase price on it, to make sure that the students can visualize what the cost will be.
- Determine the directions that you will want to deliver to your students. The Student directions listed below will not work for all students, because lower level students may need more direction than upper level students.

Student Directions:

- Read the scenario, and complete the table by combining the appropriate number of Hershey Bars and M&Ms. Remember that the total number of items should always be six.
- Write down each combination in the correct row of the given table.
- Complete the rest of the worksheet, and wait for the teacher to start the lesson.

Name: _____
Date: _____

Candy is Coming!!!

Ms. Smith decided to purchase M&M Bags and Hershey Bars for all of her students. Each M&M Bag costs $3.00, while each Hershey Bar costs $2.00. She ended up spending $16.00 on her purchase of 6 items.

1. Using the candy, complete the following table:

Number of M&M Bags	Number of Hershey Bars	Total Cost for the 6 Items ($)
0		
1		
2		
3		
4		
5		
6		

2. Circle the Row that has the Total Cost of $16.00.

3. How many Bags of M&Ms did Ms. Smith purchase?

4. How many Hershey Bars did Ms. Smith Purchase?

COMPLETE THIS SIDE WITH THE TEACHER!!

Ms. Smith decided to purchase M&M Bags and Hershey Bars for all of her students. Each M&M bag costs $3.00, while each Hershey bar costs $2.00. She ended up spending $16.00 on her purchase of 6 items.

- Define your variables.

- Write a system of equations to model the above situation.

- How many M&M Bags did Ms. Smith Purchase?

- How many Hershey Bars did Ms. Smith Purchase?

Name: *Answer Key*
Date: _____

Candy is Coming!!!

Ms. Smith decided to purchase M&M Bags and Hershey Bars for all of her students. Each M&M Bag costs $3.00, while each Hershey Bar costs $2.00. She ended up spending $16.00 on her purchase of 6 items.

1. Using the candy, complete the following table:

Number of M&M Bags	Number of Hershey Bars	Total Cost for the 6 Items ($)
0	6	12
1	5	13
2	4	14
3	3	15
4	2	16
5	1	17
6	0	18

2. Circle the Row that has the Total Cost of $16.00.

   Circle is above on the Table.

3. How many Bags of M&Ms did Ms. Smith purchase?

   Ms. Smith Purchased 4 Bags of M&Ms.

4. How many Hershey Bars did Ms. Smith Purchase?

   Ms. Smith Purchased 2 Hershey Bars.

COMPLETE THIS SIDE WITH THE TEACHER!!

Ms. Smith decided to purchase M&M Bags and Hershey Bars for all of her students. Each M&M bag costs $3.00, while each Hershey bar costs $2.00. She ended up spending $16.00 on her purchase of 6 items.

- Define your variables.

  $x =$ Number of M&M Bags Purchased
  $y =$ Number of Hershey Bars Purchased

- Write a system of equations.

  Cost: $3.00x + 2.00y = 16.00$
  Items: $x + y = 6$

- How many M&M Bags did Ms. Smith Purchase?

  Finding M&M Bags means you should eliminate y.

$$\begin{array}{rcl} & & 3x + 2y = 16.00 \\ 3x + 2y = 16.00 & \xrightarrow{1} & -2x - 2y = -12 \\ \hline x + y = 6 & \xrightarrow{-2} & x = 4 \end{array}$$

  Ms. Smith Purchased 4 Bags of M&Ms.

- How many Hershey Bars did Ms. Smith Purchase?

  Since we have already solved for x, we can just substitute $x = 4$ back into both equations.

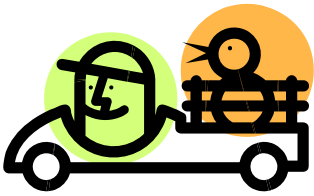
$$\begin{array}{r} 3.00x + 2.00y = 16.00 \\ 3(4) + 2y = 16.00 \\ 12 + 2y = 16 \\ \hline -12 \qquad -12 \\ \hline \frac{2y}{2} = \frac{4}{2} \\ y = 2 \end{array}$$

  Ms. Smith purchase 2 Hershey Bars.

Standard Form Real World Model Problems

Name: _____
Date: _____

1. Old McDonald had a farm that had Chickens and Ducks. Everyday Mr. McDonald collects 19 eggs, and he knows that each Duck lays 2 eggs, while each Chicken lays 3 eggs. But each week, every Duck eats 3 pounds of feed, while every chicken eats 4 pounds of feed, for a total of 26 pounds of feed.



- What do the two variables in this system represent?

- Write a system of equations to represent the model.

- How many ducks are there? How many chickens are there? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

2. At the local Convenience store William and Sarah are getting snacks for the friends. William buys 3 soft drinks and 2 hot dogs at a cost of $ 7.70, while Sarah buys 2 soft drinks and 1 hot dog at cost of $ 4.55.

- What do the two variables in this system represent?

- Write a system of equations to represent the model.

- What is the cost of 1 soft drink? What is the cost of 1 hot dog? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

If you felt as though you got #1 and 2 correct, go to Problem #4. If you feel as though you need extra help go to Question 3 and do not complete Question #4.

3. Last weekend, the Knights of the Round Table held a Jousting contest. During the contest, each knight had 3 spears, and each squire had 2 spears, for a total of 32 spears. Also, each knight had 2 swords, and each squire had only 1 sword, for a total of 19 swords.

   - What do the two variables in this system represent? (Hint: You are trying to figure out how many Knights and How many Squires were at the Joust.)

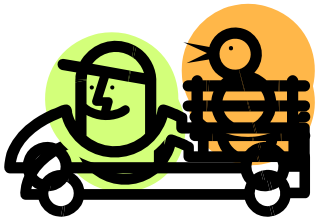
   - Write a system of equations to represent the model. (Hint: Go to the scenario and every place you see the word Knight or Squire replace it with the variables that you chose. Also, think about what the word "Total" means in Math.)

   - How many knights are there? How many squires are there? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation. (Hint: Remember this means to solve the equation.)

4. Instead of completing another problem, be creative and write your own scenario. Be sure to give your solution as well. (Hint: The easiest way to come up with this is to determine your answer first.)

Standard Form Real World Model Problems

Name: *Answer Key*
Date: _____



1. Old McDonald had a farm that had Chickens and Ducks. Everyday Mr. McDonald collects 19 eggs, and he knows that each Duck lays 2 eggs, while each Chicken lays 3 eggs. But each week, every Duck eats 3 pounds of feed, while every chicken eats 4 pounds of feed, for a total of 26 pounds of feed.

   - What do the two variables in this system represent?

     c = number of chickens

     d = number of ducks

   - Write a system of equations to represent the model.

     (eggs) $2d + 3c = 19$

     (feed) $3d + 4c = 26$

   - How many ducks are there? How many chickens are there? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

StepA

$(-3)(2d + 3c) = (19)(-3)$
$(2)(3d + 4c) = (26)(2)$

$-6d - 9c = -57$
$6d + 8c = 52$

$-d = -5$

$(-1)(-d) = (-1)(-5)$

$d = 5$ ducks

Step B

$2(5) + 3c = 19$
$10 + 3c = 19$

$10 + 3c = 19$
$-10 \quad -10$
$3c = 9$

$\frac{3c}{3} = \frac{9}{3}$

$c = 3$ chickens

There are 5 ducks and 3 chickens on the farm.

2. At the local Convenience store William and Sarah are getting snacks for the friends. William buys 3 soft drinks and 2 hot dogs at a cost of $ 7.70, while Sarah buys 2 soft drinks and 1 hot dog at cost of $ 4.55.

- What do the two variables in this system represent?

  d = cost of a soft drink

  h = cost of a hot dog

- Write a system of equations to represent the model.

  (William) 3d + 2h = 7.7

  (Sarah) 2s + h = 4.55

- What is the cost of 1 soft drink? What is the cost of 1 hot dog? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation.

Step A $3d + 2h = 7.7$

$(-2)(2s + h) = 4.55(-2)$

$3d + 2h = 7.7$

$-4d - 2h = -9.1$

$-d = -1.4$

$(-1)(-d) = (-1)(-1.4)$

$d = \$ 1.40$

Step B $2(1.4) + h = 4.55$

$2.8 + h = 4.55$

$-2.8 \quad -2.8$

$h = \$ 1.75$

The cost of soft drink is $ 1.40 and the cost of a hot dog is $ 1.75.

If you felt as though you got #1 and 2 correct, go to Problem #4. If you feel as though you need extra help go to Question 3 and do not complete Question #4.

3. Last weekend, the Knights of the Round Table held a Jousting contest. During the contest, each knight had 3 spears, and each squire had 2 spears, for a total of 32 spears. Also, each knight had 2 swords, and each squire had only 1 sword, for a total of 19 swords.

- What do the two variables in this system represent? (Hint: You are trying to figure out how many Knights and how many Squires were at the Joust.)

  k = number of knights
  s = number squires

- Write a system of equations to represent the model. (Hint: Go to the scenario and every place you see the word Knight or Squire replace it with the variables that you chose. Also, think about what the word "Total" means in Math.)

  (spears) $3k + 2s = 32$

  (swords) $2k + s = 19$

- How many knights are there? How many squires are there? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation. (Hint: Remember this means to solve the equation.)

Step A

$$3k + 2s = 32$$
$$(-2)(2k + s) = (19)(-2)$$

$$3k + 2s = 32$$
$$-4k - 2s = -38$$
$$-k = -6$$
$$(-1)(-k) = (-1)(-6)$$
$$k = 6 \text{ knights}$$

Step B

$$2(6) + s = 19$$
$$12 + s = 19$$
$$-12 \quad -12$$
$$s = 7 \text{ squires}$$

There are 6 knight and 7 squires.

4. Instead of completing another problem, be creative and write your own scenario. Be sure to give your solution as well. (Hint: The easiest way to come up with this is to determine your answer first.)

Answers will vary based on what the students put as their answers.

Warm-Up Activity

Name: _____
Date: _____

For Questions #1 and #2, match the answer with the question.

1. Linear System ____ **A.** $\begin{cases} 3x+5y=17 \\ 7x+4y=63 \end{cases}$

2. Standard System ____ **B.** $\begin{cases} y=17x+89 \\ y=39x+42 \end{cases}$

For Questions #3 and #4, complete the sentence with the best answer.

3. In $y=mx+b$ real world linear systems, there are always two _____

   and there are always two_____.

4. A standard linear system has two equations in the

   form, and each equation has a _____.

For Question #5, answer the question.

5. How would you tell the difference between a Slope-Intercept Form ($y=mx+b$) system and a Standard From ($Ax+By=C$) Linear System in a real world problem?

_____

_____

_____

Warm-Up Activity

Name: *Answer Key*
Date: _____

For Questions #1 and #2, match the answer with the question.

1. Linear System *B*

A. $\begin{cases} 3x+5y=17 \\ 7x+4y=63 \end{cases}$

2. Standard System *A*

B. $\begin{cases} y=17x+89 \\ y=39x+42 \end{cases}$

For Questions #3 and #4, complete the sentence with the best answer.

3. In $y=mx+b$ real world linear systems, there are always two *rates of change* and there are always two *initial conditions or amounts of what you start with.*

4. A standard linear system has two equations in the $Ax+By=C$ form, and each equation has a *numeric total*.

For Question #5, answer the question.

5. How would you tell the difference between a Slope-Intercept Form ($y=mx+b$) system and a Standard From ($Ax+By=C$) Linear System in a real world problem?

   Look for either a problem with two totals or a problem with two rates of change and two initial conditions.

Mix-and-Match Real World Systems of Equations Activity
(Each Student Presents to the class)

Teacher Notes

Materials:

- Copy of the "Real World Systems of Equations Index Cards"
- Prepared Set of Index Cards, one card for each student in the class
- Full Copy of this Activity, one for just the teacher.

Preparation:

- Prepare the index cards. On each index card, place either the real world scenario or the system of equations. (There will be nothing on the back of the cards.) You can find questions attached to this activity.
- Determine the directions that you will want to deliver to your students. The Student directions listed below will not work for all students, because lower level students may need more direction than upper level students.

Student Directions:

- Each student will receive one index card, either a real world scenario or the system of equations.
- The student's goal is to find the matching index card for their own index card. For example, if I would have the following Scenario Card:

Ms. Smith decided to purchase M&M Bags and Hershey Bars for all of her students. Each M&M Bag costs $3, while each Hershey Bar Cost $2. She ended up spending $16 on her purchase of 6 items.

I would be attempting to find the student that had the following Systems of Equations Card:

$$3x + 2y = 16$$
$$x + y = 6$$

- Once each student has found their partner, they are to prepare a presentation for the class. In this presentation, they must be able to explain why their cards match and how they determined the answer to this real world situation.
- During the presentations, it is the listener's responsibility to ask questions about the problem and to give suggestions on how they could improve their answers.

This Activity was modified from "*Cooperative Learning & Mathematics High School Activities*," Kagan Publishing, 2001. The original Activity is called "Mix-and-Match."

Mix-and-Match Real World Systems of Equations Activity
(Each Pair makes a Poster and the Class Performs a Merry-Go-Round)

Teacher Notes

Materials:

- Copy of the "Real World Systems of Equations Index Cards"
- Prepared Set of Index Cards, one card for each student in the class
- Poster Size Paper, one for each pair of students
- Full Copy of this Activity, one for just the teacher.

Preparation:

- Prepare the index cards. On each index card, place either the real world scenario or the system of equations. (There will be nothing on the back of the cards.) You can find questions attached to this activity.
- Determine the directions that you will want to deliver to your students. The Student directions listed below will not work for all students, because lower level students may need more direction than upper level students.

Student Directions:

- Each student will receive one index card, either a real world scenario or the system of equations.
- The student's goal is to find the matching index card for their own index card. For example, if I would have the following Scenario Card:

Ms. Smith decided to purchase M&M Bags and Hershey Bars for all of her students. Each M&M Bag costs \$3, while each Hershey Bar Cost \$2. She ended up spending \$16 on her purchase of 6 items.

I would be attempting to find the student that had the following Systems of Equations Card:

$$3x + 2y = 16$$
$$x + y = 6$$

- Once each student has found their partner, they are to prepare a poster for the class. On this poster, they must have an explanation as to why their cards match and how they determined the answer to this real world situation.
- After the class has prepared their posters, students will go around the room writing comments/concerns about each poster. They can either write directly on the poster, or you can provide them with Sticky Notes. Coaching maybe necessary to get the best possible responses from each of the students and to prevent unnecessary or derogatory responses.

Real World Systems of Equations Index Cards

1. Marcus and his friends are having a Final Four party. Marcus buys 5 burgers and 9 sodas for $28.75 at the Beefy Burger joint. When five more of his friends show up, he heads back to Beefy Burger for 10 more burgers and 10 more sodas and spends $47.50.	$5x + 9y = 28.75$ $10x + 10y = 47.50$
2. You buy 5 bags of chips and 9 bags of pretzels for $16.95. Later you buy 10 bags of chips and 10 bags of pretzels for $25.50.	$5x + 9y = 16.95$ $10x + 10y = 25.50$
3. You empty your coin jar and find 49 coins (all nickels and quarters). The total value of the coins is $8.65.	$x + y = 49$ $.05x + .25y = 8.65$
4. You dump the 49 coins from your piggy bank into a CoinstarCountemUp machine. You have only dimes and quarters which the CoinstarCountemUp machine counts as $7.45.	$x + y = 49$ $.10x + .25y = 7.45$

Real World Systems of Equations Index Cards

5. An army of goblins and orcs eats breakfast before a busy day of burning and pillaging. Each orc eats 4 pieces maggot bread while each goblin eats 3 pieces of maggot bread. The army eats 265 pieces of maggot bread. Each orc has 3 weapons. Each goblin has weapons. The army has 190 weapons.	$4x + 3y = 265$ $3x + 2y = 190$
6. The "Pay Upfront" car rental company charges an insurance and gas fee of \$33 and daily rental fee of \$8 per day. The "Drive Now Pay Later" car rental company charges an insurance and gas fee of \$18 and has a daily rental fee of \$11 per day.	$y = 8x + 33$ $y = 11x + 18$
7. Katherine is deciding which catering company she should use for Mom's 50^{th} birthday party. At Bashful's Birthday Blast there is a fee of \$16.95 and a charge of \$9.00 per person. At Bertha's Ballroom Blitz there is a fee of \$25.50 and a charge of \$5.00 per person.	$y = 9x + 16.95$ $y = 5x + 25.50$
8. Farmer Peter and Farmer Paula are picking apples from their apple trees. Farmer Peter as already picked 287 apples. Farmer Peter picks another 5 apples each minute. Farmer Paula has already picked 154 apples. Farmer Paula picks another 6 apples each minute.	$y = 5x + 287$ $y = 6x + 154$

Real World Systems of Equations Index Cards

9. Jill bought one hot dog and two soft drinks for a cost of $ 4.95. Jack bought three hot dogs and one soft drink for a cost of $ 6.85.	$x + 2y = 4.95$ $3x + y = 6.85$
10. There are a total of 34 lions and hyenas. Each lion eats 4 antelope. Each hyena eats 3 antelope. 117 antelope are eaten.	$x + y = 34$ $4x + 3y = 117$
11. Two companies sell stock shares on Wall Street. Losers Incorporated stock starts with a value of 39.63 and loses $0.08 for each new stockholder. The "Be Rich with Us" Corporation starts with a value of $24.45 and gains $0.03 in value for each new stockholder.	$y = -.08x + 39.63$ $y = 0.03x + 24.45$
12. Two countries are keeping track of the cost of a barrel of oil in their country. In the country, Weneedaoil, a barrel of oil costs of $40.05 and the price increases $0.08 each day. In the country, Wehaveaoil, a barrel of oil costs of $48.74 and the price decreases $0.03 each day.	$y = 0.08x + 40.05$ $y = -.03x + 48.74$

Real World Systems of Equations Index Cards

13. The cats Killer and Croaker are catch mice on the waterfront. Killer has already caught 29 mice and catches 87 more mice each month. Croaker has already caught 282 mice and catches 64 more mice each month.	$y = 87x + 29$ $y = 64x + 282$
14. Ted and Ara are saving money. Ara has saved $1,477 and saves 64 more dollars each payday. Ted has saved $948 and saves 87 more dollars each pay day.	$y = 87x + 948$ $y = 64x + 1477$
15. Each month, the Weovabillya cell phone company has an operating fee of $28.55 and charges $0.48 per minute of calls. Each month, the Dropyocall Corporation has an operating fee of $59 and charges $0.19 per minute of calls.	$y = 0.19x + 59$ $y = 0.48 + 28.55$
16. The Slow Fix auto shop charges $28 for parts and $48 per hour of labor. The We Work Cheaper auto shop charges $59 for parts and $44.90 per hour of labor.	$y = 48x + 28$ $y = 44.9x + 59$

Real World Systems of Equations Index Cards

Answer Key

1. Marcus and his friends are having a Final Four party. Marcus buys 5 burgers and 9 sodas for $28.75 at the Beefy Burger joint. When five more of his friends show up, he heads back to Beefy Burger for 10 more burgers and 10 more sodas and spends $47.50.	$5x + 9y = 28.75$ $10x + 10y = 47.50$ Burgers cost $3.50 and Sodas cost $1.25.
2. You buy 5 bags of chips and 9 bags of pretzels for $16.95. Later you buy 10 bags of chips and 10 bags of pretzels for $25.50.	$5x + 9y = 16.95$ $10x + 10y = 25.50$ Chips cost $1.50 and Pretzels cost $1.05.
3. You empty your coin jar and find 49 coins (all nickels and quarters). The total value of the coins is $8.65.	$x + y = 49$ $0.05x + 0.25y = 8.65$ There are 18 nickels and 31 quarters.
4. You dump the 49 coins from your piggy bank into a Coin-star Count'em Up machine. You have only dimes and quarters which the Coin-star Count'em Up machine counts as $7.45.	$x + y = 49$ $.10x + .25y = 7.45$ There are 32 dimes and 17 quarters.

Real World Systems of Equations Index Cards

Answer Key

5. An army of goblins and orcs eats breakfast before a busy day of burning and pillaging. Each orc eats 4 pieces maggot bread while each goblin eats 3 pieces of maggot bread. The army eats 265 pieces of maggot bread. Each orc has 3 weapons. Each goblin has 2 weapons. The army has 190 weapons.	$4x + 3y = 265$ $3x + 2y = 190$ There are 40 Goblins and 35 Orcs.
6. The "Pay Upfront" car rental company charges an insurance and gas fee of \$33 and daily rental fee of \$8 per day. The "Drive Now Pay Later" car rental company charges an insurance and gas fee of \$18 and has a daily rental fee of \$11 per day.	$y = 8x + 33$ $y = 11x + 18$ At 5 days, the cost will be \$73.
7. Katherine is deciding which catering company she should use for Mom's 50^{th} birthday party. At Bashful's Birthday Blast there is a fee of \$20.00 and a charge of \$9.00 per person. At Bertha's Ballroom Blitz there is a fee of \$100 and a charge of \$5.00 per person.	$y = 9x + 20$ $y = 5x + 100$ At 20 People, it will cost \$200.

Real World Systems of Equations Index Cards

Answer Key

8. Farmer Peter and Farmer Paula are picking apples from their apple trees. Farmer Peter as already picked 287 apples. Farmer Peter picks another 5 apples each minute. Farmer Paula has already picked 154 apples. Farmer Paula picks another 6 apples each minute.	$y = 5x + 287$ $y = 6x + 154$ At 133 minutes, they will have picked 952 apples.
9. Jill bought one hot dog and two soft drinks for a cost of $ 4.95. Jack bought three hot dogs and one soft drink for a cost of $ 6.85.	$x + 2y = 4.95$ $3x + y = 6.85$ A hot dog will costs $1.75 and a soft drink costs $1.60.
10. There are a total of 34 lions and hyenas. Each lion eats 4 antelope. Each hyena eats 3 antelope. 117 antelope are eaten.	$x + y = 34$ $4x + 3y = 117$ There are 15 lions and 19 hyenas.
11. Two companies sell stock shares on Wall Street. Losers Incorporated stock starts with a value of $39.63 and loses $0.08 for each new stockholder. The "Be Rich with Us" Corporation starts with a value of $24.45 and gains $0.03 in value for each new stockholder.	$y = -.08x + 39.63$ $y = 0.03x + 24.45$ At 138 shareholders, the cost will be $28.59 per share.

Real World Systems of Equations Index Cards

Answer Key

12. Two countries are keeping track of the cost of a barrel of oil in their country. In the country, Weneedaoil, a barrel of oil costs of $40.05 and the price increases $0.08 each day. In the country, Wehaveaoil, a barrel of oil costs of $48.74 and the price decreases $0.03 each day.	$y = 0.08x + 40.05$ $y = -.03x + 48.74$ At 79 days, the cost will be $46.37.
13. The cats Killer and Croaker are catch mice on the waterfront. Killer has already caught 29 mice and catches 87 more mice each month. Croaker has already caught 282 mice and catches 64 more mice each month.	$y = 87x + 29$ $y = 64x + 282$ At 11 months, there will be 986 mice.
14. Ted and Ara are saving money. Ara has saved $1,477 and saves 64 more dollars each payday. Ted has saved $948 and saves 87 more dollars each pay day.	$y = 87x + 948$ $y = 64x + 1477$ At 23 paydays, they both will be $2949.

Real World Systems of Equations Index Cards

Answer Key

15. Each month, the Weovabillya cell phone company has an operating fee of $28.55 and charges $0.48 per minute of calls. Each month, the Drop-Yo-Call Corporation has an operating fee of $59 and charges $0.19 per minute of calls.	$y = 0.19x + 59$ $y = 0.48x + 28.55$ At 105 minutes, the cost will be $78.95.
16. The Slow Fix auto shop charges $28 for parts and $48 per hour of labor. The We Work Cheaper auto shop charges $59 for parts and $44.90 per hour of labor.	$y = 48x + 28$ $y = 44.9x + 59$ At 10 hours they will cost $508.

Real World Linear System Quiz **Name: _____**
Date: _____

17 cars and trucks get gasoline at the gas station. Each car gets 8 gallons of gasoline. Each truck gets 19 gallons of gasoline. The station sells 169 gallons of gasoline.

1. What does each of the variables represent? (10 points)

2. Write a system of equations for above situation. (15 points)

3. What is the number of trucks? What is the number of cars? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation. (20 points)

The We-Are-Readers Book Club allows members to buy books for $ 8.75 per book. The Print-Is-Dead Book Club costs $ 20 to join and allows members to buy books for $ 6.25 per book.

4. What does each of the variables represent? (10 points)

5. Write a system of equations for above situation. (15 points)

6. How many book purchases give the two book clubs the same cost? What is the equal cost? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation. (25 points)

Real World Linear System Quiz

Name: ***Answer Key***
Date: _____

17 cars and trucks get gasoline at the gas station. Each car gets 8 gallons of gasoline. Each truck gets 19 gallons of gasoline. The station sells 169 gallons of gasoline.

1. What does each of the variables represent? (10 points)

   t = number of trucks

   c = number of cars

   5 points for each correct answer

   Full credit can be awarded for this question if a student's later answers reflect a full understanding of the variables' meaning using units or sentence that reflect an understanding of the variable's meaning.

2. Write a system of equations for above situation. (15 points)

   c + t = 17

   8c + 19t = 169

   5 points for showing recognition that equation(s) should be in standard form

   5 points for first correct equation

   5 points for second correct equation

3. What is the number of trucks? What is the number of cars? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation. (20 points)

Step 1 $(-8)(c + t) = 17(-8)$

$8c + 19t = 169$

5 points for showing a valid method that is either incomplete or has significant error(s)

Step 2 $-8c - 8t = -136$

$\underline{8c + 19t = 169}$

$11t = 33$

5 points if method is fully correct or has only minor error(s) that do not reflect a lack of proficiency

Step 3 $\frac{11t}{11} = \frac{33}{11}$

First Answer $t = 3$ trucks

5 points Answer is correct with the Proper units or earlier variable definition

Step 4 $c + 3 = 17$

Step 5 $c + 3 = 17$
$-3 \quad -3$

Second Answer $c = 14$

5 points for showing a valid method that is correctly followed either correctly or has only minor error(s)

5 points Answer is correct with the proper units or earlier variable definitions

The WeAreReaders book club allows members to buy books for $ 8.75 per book. The PrintIsDead book club costs $ 20 to join and allows members to buy books for $ 6.25 per book.

4. What does each of the variables represent? (10 points)

b = number of books

c = cost in dollars

5 points for each correct answer

Full credit can be awarded for this question if a student's later answers reflect a full understanding of the variables meaning using units or sentence that reflect an understanding of the variable's meaning.

5. Write a system of equations for above situation. (15 points)

c = 8.75b

c = 6.25b + 20

5 points for showing recognition that equation(s) should be in slope intercept form

5 points for first correct equation
5 points for second correct equation

6. How many book purchases give the two book clubs the same cost? What is the equal cost? Use mathematics to explain how you determined your answer. Use words, symbols or both in your explanation. (25 points)

Step 1 $8.75b = 6.25b + 20$

5 points for showing a valid method that is either incomplete or has a significant error

Step 2

$$\begin{array}{r} 8.75b = 6.25b + 20 \\ \underline{-6.25b \quad -6.25b} \\ 2.5b = 20 \end{array}$$

5 points if method is fully correct or has only minor errors that do not reflect a lack of proficiency

Step 3 $\frac{2.5b}{2.5} = \frac{20}{2.5}$

5 points Answer is correct with the proper units or earlier variable definition

First Answer $b = 8$ books

Step 4 $c = 8.75 (8)$

5 points for showing a valid method that is correctly followed or has only minor errors

Step 5 $c = \$ 70$

5 points Answer is correct with the proper units or earlier variable definitions